

$B_{s,d}-\bar{B}_{s,d}$ mixings and $b \rightarrow q(\gamma, l\bar{l})$ decays in private Higgs model

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We study the low energy phenomena induced by the lightest charged Higgs in the private Higgs model, in which each quark flavor is associated with a Higgs doublet. We show that the couplings of the charged Higgs scalars to fermions are fixed and the unknown parameters are only the masses and mixing elements of the charged Higgs scalars. As the charged Higgs masses satisfy with $M_b < M_c \ll M_s \ll M_{d,u}$, processes involving B -meson are expected to be the ideal places to test the private Higgs model. In particular, we explore the constraints on the model from experimental data in B physics, such as the branching ratio (BR) and CP asymmetry of $B \rightarrow X_s \gamma$, $B_{d,s}-\bar{B}_{d,s}$ mixings and the BR for $B \rightarrow K^* \ell^+ \ell^-$. We illustrate that the sign of the Wilson coefficient for $B \rightarrow X_s \gamma$ can be different from that in the standard model, while this flipped sign can be displayed by the forward-backward asymmetry of $B \rightarrow V \ell^+ \ell^-$ with V a vector meson. We also demonstrate that $B_{d,s}-\bar{B}_{d,s}$ mixings and their time-dependent CP asymmetries are negligible small and the BR of $B \rightarrow K^* \ell^+ \ell^-$ can have a more strict bound than that of $B \rightarrow X_s \gamma$.

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I. INTRODUCTION

The masses of quarks and charged leptons are dictated by the Yukawa sector in the standard model (SM) through the simple and elegant Higgs mechanism, where the vacuum expectation value (VEV) of the Higgs field, determined by massive gauge bosons, indicates the scale of the electroweak symmetry breaking (EWSB) with $\langle H \rangle = v/\sqrt{2} = 246$ GeV. According to the data, there appear mass hierarchies in the generations of charged fermions such as $m_u \ll m_c \ll m_t$, $m_d \ll m_s \ll m_b$, and $m_e \ll m_\mu \ll m_\tau$, while $m_t \gg m_b$ and $m_c \gg m_s$ but $m_u < m_d$ [1]. In the SM, due to the scale of the EWSB being fixed by the VEV of the Higgs field, the mass hierarchies are ascribed to the fine-tuning of the Yukawa couplings.

In the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2], defined by $V_{\text{CKM}} = V_L^U V_L^{D\dagger}$ with the unitary matrix $V_L^{U(D)}$ for diagonalizing the quark mass matrices, it is known that the off-diagonal elements denoted by $(V_{\text{CKM}})_{i \neq j}$ are suppressed by the Wolfenstein parameter λ [3]. If the effects of λ are turned off, one immediately finds $V_L^U = V_L^D$. In other words, small elements of $(V_{\text{CKM}})_{i \neq j}$ imply that the structures of the Yukawa matrices for up and down type quarks should be close to each other. However, based on the above discussion, the similarity of the mass structures is not respected by the data. Plausibly, we need to extend the Yukawa sector to explain the mass hierarchies.

In order to evade the drawback of the fine-tuned Yukawa couplings, a new type of solutions to the mass hierarchy is recently proposed in Refs. [4,5], in which the authors extend one Higgs doublet in the SM to multi-Higgs doublets with each gauge singlet right-handed fermion associated with one Higgs doublet. Hereafter, the model is called the private Higgs (PH) model [4]. The philosophy of solving the mass hierarchies in generations is now to utilize the hierarchy of VEVs of scalar fields instead of the hierarchy of the Yukawa couplings. Although many new neutral and charged scalar bosons are introduced in the PH model, most of the effects are suppressed by the heavy masses. In addition, the PH model provides the candidate of dark matter. The detailed study could be referred to Ref. [6].

Since top and bottom quarks are the first two heaviest fermions, the dominant new effects are expected to be associated with the Higgs doublets, denoted by $\Phi_{t,b}$, respectively. Since $m_t \gg m_b$ implies $\langle \Phi_t \rangle \gg \langle \Phi_b \rangle$, Φ_b gives the dominant new physical effects if we take Φ_t as the SM Higgs. Accordingly, we anticipate that the B -meson system could be a good environment to probe the special character in the PH model. In this paper, we study the effects of the private charged Higgs bosons on the rare flavor changing neutral current (FCNC) processes, such as $B_q-\bar{B}_q$ mixings and $b \rightarrow q(\gamma, \ell^+ \ell^-)$ decays with $q = s, d$. These processes are expected to be sensitive to the charged Higgs sector.

The paper is organized as follows. In Sec. II, we briefly summarize the PH model. In Sec. III, we study the contributions of the charged Higgs scalars on $B_q-\bar{B}_q$ mixings, $B \rightarrow X_s \gamma$, $B_q \rightarrow \ell^+ \ell^-$, and $B \rightarrow (P, V) \ell^+ \ell^-$ decays. The

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numerical results and their discussions are given in Sec. IV. Finally, we present the summary in Sec. V.

II. CHARGED HIGGSSES IN PRIVATE HIGGS MODEL

To examine the charged-Higgs effects in the PH model, we first review the model proposed in Ref. [4]. In the model, as the hierarchy of the scalar VEVs is used to understand the fermion masses instead of the arbitrary Yukawa couplings, the SM with one Higgs doublet is extended to include six Higgs doublets so that each Higgs doublet can only couple to one flavor with imposing a set of six Z_2 discrete symmetries. In addition, six gauge singlet real scalars are introduced to achieve the spontaneous symmetry breakings. For simplicity, we will take only one singlet scalar field S in our discussion. The six-singlet case can be easily accommodated but our results on FCNCs remain the same. Under the discrete symmetries, the transformations for the flavor and the scalars are set to be

$$f_R \rightarrow -f_R, \quad \Phi_f \rightarrow -\Phi_f, \quad S \rightarrow -S, \quad (1)$$

where f denotes the possible flavor of the quark, Φ_f is the associated Higgs doublet scalar and S is the gauge singlet scalar. Since the left-handed quark belongs to the $SU(2)$ doublet of two flavors, we require that it is invariant under the discrete transformations. Accordingly, the related scalar interactions with the electroweak gauge and Z_2 discrete symmetries are given by [4]

$$\begin{aligned} \mathcal{L} = & \partial_\mu S \partial^\mu S - \frac{\lambda_s}{4} \left(S^2 - \frac{v_s^2}{2} \right)^2 + \sum_f \left[(D_\mu \Phi_f)^\dagger (D^\mu \Phi_f) \right. \\ & - \frac{1}{2} M_f^2 \Phi_f^\dagger \Phi_f - \lambda_f (\Phi_f^\dagger \Phi_f)^2 + g_{sf} S^2 \Phi_f^\dagger \Phi_f \left. \right] \\ & + \sum_{f \neq f'} \left[\frac{\gamma_{ff'}}{\sqrt{2}} v_s S \Phi_f^\dagger \Phi_{f'} + a_{ff'} \Phi_f^\dagger \Phi_{f'} \Phi_f^\dagger \Phi_{f'} \right. \\ & \left. + b_{ff'} \Phi_f^\dagger \Phi_f \Phi_{f'}^\dagger \Phi_{f'} + c_{ff'} \Phi_f^\dagger \Phi_{f'} \Phi_{f'}^\dagger \Phi_f \right] - \mathcal{L}_Y, \end{aligned} \quad (2)$$

where $D_\mu = i\partial_\mu - g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - g' \frac{Y}{2} B_\mu$ is the covariant derivative, M_f is the mass of Φ_f , v_s is the VEV of S , v_x is a free parameter, the values of $\gamma_{ff'}$, $a_{ff'}$, $b_{ff'}$, and $c_{ff'}$ are regarded as the same order of magnitude, and \mathcal{L}_Y stands for the Yukawa sector to be given. Since the top quark is the heaviest quark with its mass close to the EWSB scale, it is natural to take Φ_t as the Higgs doublet in the SM. Therefore, to develop a nonzero VEV of Φ_t to have the EWSB spontaneously, the condition of $M_t^2/2 < g_{st} v_s^2$ should be satisfied. Consequently, the relevant scalar potential with the leading terms is given by

$$V^{\text{LT}} = \frac{\lambda_s}{4} \left(S^2 - \frac{v_s^2}{2} \right)^2 + \lambda_t (\Phi_t^\dagger \Phi_t)^2 - g_{st} S^2 \Phi_t^\dagger \Phi_t. \quad (3)$$

By minimizing Eq. (3), the VEVs of S and Φ_t are obtained as

$$\langle S \rangle^2 \equiv \frac{v_s^2}{2} = \frac{1}{2} \frac{\lambda_s \lambda_t}{\lambda_s \lambda_t - g_{st}^2} v_x^2, \quad \langle \Phi_t^0 \rangle^2 \equiv \frac{v_t^2}{2} = \frac{g_{st}}{2\lambda_t} v_s^2. \quad (4)$$

We now discuss how to get the small VEVs for $\Phi_{f \neq t}$. Unlike the case for Φ_t , we need to adopt the condition $M_f > \sqrt{g_{sf}} v_s$ for $f \neq t$. The relevant subleading scalar potential for $f \neq t$ is

$$V^{\text{SLT}} = \sum_{f \neq t} \left[\frac{1}{2} M_f^2 \Phi_f^\dagger \Phi_f - \left(\frac{\gamma_{tf}}{\sqrt{2}} v_s S \Phi_t^\dagger \Phi_f + \text{H.c.} \right) \right]. \quad (5)$$

We note that although the coefficients of $a_{tf'}$, $b_{tf'}$ and $c_{tf'}$ are similar to $\gamma_{tf'}$ in magnitude, their effects are sub-subleading and negligible due to the associated VEVs of scalar fields being much less than v_s . Similarly, by minimizing Eq. (5) the VEV of $\Phi_{f \neq t}^0$ is given by [4]

$$\langle \Phi_f^0 \rangle = \gamma_{tf} \frac{v_t}{\sqrt{2}} \frac{v_s^2}{2M_f^2}. \quad (6)$$

Clearly, if we set γ_{tf} to be the same order of magnitude for a different f , the hierarchy of VEVs could be obtained by controlling M_f , i.e., the heavier M_f is, the smaller $\langle \Phi_f^0 \rangle$ will be for $f \neq t$.

After introducing the strategy to obtain the EWSB spontaneously as well as the small VEVs of the scalar fields with $f \neq t$, we can proceed to investigate the characters of the charged Higgs scalars in the PH model. In terms of $SU(2)_L \times U(1)_Y$ gauge symmetries, the Yukawa sector is given by

$$\mathcal{L}_Y = -\bar{Q}'_L Y_D \Phi_D d'_R - \bar{Q}'_L Y_U \tilde{\Phi}_U u'_R + \text{H.c.}, \quad (7)$$

where $Q'_L = (u', d')_L$ and q'_R denote the doublet and singlet of $SU(2)_L$, respectively, and $Y_{D(U)}$ is the 3×3 Yukawa matrix for down (up) type quarks. In the flavor space, $\Phi_{D,U}$ are also 3×3 matrices, given by

$$\Phi_D = \begin{pmatrix} \Phi_d & 0 & 0 \\ 0 & \Phi_s & 0 \\ 0 & 0 & \Phi_b \end{pmatrix}, \quad \tilde{\Phi}_U = \begin{pmatrix} \tilde{\Phi}_u & 0 & 0 \\ 0 & \tilde{\Phi}_c & 0 \\ 0 & 0 & \tilde{\Phi}_t \end{pmatrix}, \quad (8)$$

where $\Phi_D^T = (\phi^+, \phi^0)_D$ and $\tilde{\Phi}_U = i\tau_2 \Phi_U^*$ are the Higgs doublets of $SU(2)_L$, which couple to $D = (d, s, b)$ and $U = (u, c, t)$, respectively. After the EWSB with the shifted scalar fields

$$\phi_F^0 = \frac{1}{\sqrt{2}}(v_F + H_F + iA_F), \quad (F = D, U),$$

the mass terms of quarks in the Yukawa sector are developed to be

$$\mathcal{L}_{\text{mass}} = -\bar{d}'_L Y_D \frac{\mathbf{V}_D}{\sqrt{2}} d'_R - \bar{u}'_L Y_U \frac{\mathbf{V}_U}{\sqrt{2}} u'_R + \text{H.c.} \quad (9)$$

with

$$\mathbf{V}_{D(U)} = \begin{pmatrix} v_{d(u)} & 0 & 0 \\ 0 & v_{s(c)} & 0 \\ 0 & 0 & v_{b(t)} \end{pmatrix}. \quad (10)$$

To avoid the large FCNCs at tree level, we adopt the Yukawa matrices in Ref. [4], given by

$$Y_Q^Q = \lambda_Q \delta_{ij} + \epsilon_{ij}^Q, \quad (11)$$

where $Q = U$ and D , $\lambda_Q \sim O(1)$ and $\epsilon^Q \ll 1$. By combining with $v_{d(u)} \ll v_{s(c)} \ll v_{b(t)}$, the quark mass matrices can be simplified as

$$\begin{aligned} M_D &= \begin{pmatrix} (\mathbf{m}_d)_{2 \times 2} & \epsilon_{d2 \times 1} \\ \mathbf{0}_{1 \times 2} & m_b \end{pmatrix}, \\ M_U &= \begin{pmatrix} (\mathbf{m}_u)_{2 \times 2} & \epsilon_{u2 \times 1} \\ \mathbf{0}_{1 \times 2} & m_t \end{pmatrix} \end{aligned} \quad (12)$$

where $\text{dia}(\mathbf{m}_{d(u)}) = \lambda_{D(U)}(v_{d(u)}, v_{s(u)})/\sqrt{2}$ correspond to the light quarks and $m_{b(t)} = \lambda_{D(U)}v_{b(t)}/\sqrt{2}$ the first two heaviest quarks, $\epsilon_{d(u)11} = \epsilon_{13}^{D(U)}v_{b(t)}/\sqrt{2}$ and $\epsilon_{d(u)21} = \epsilon_{23}^{D(U)}v_{b(t)}/\sqrt{2}$. To get the physical states, we use $V_{L,R}^D$ and $V_{L,R}^U$ to diagonalize the mass matrices, i.e. $M_U^{\text{dia}} = V_L^U M_U V_R^{U\dagger}$ and $M_D^{\text{dia}} = V_L^D M_D V_R^{D\dagger}$. The individual informations on V_L^Q and V_R^Q can be obtained by

$$\begin{aligned} M_Q^{\text{dia}} M_Q^{\text{dia}\dagger} &= V_L^Q M_Q M_Q^\dagger V_L^{Q\dagger}, \\ M_Q^{\text{dia}\dagger} M_Q^{\text{dia}} &= V_R^Q M_Q^\dagger M_Q V_R^{Q\dagger}, \end{aligned} \quad (13)$$

respectively, where

$$\begin{aligned} M_Q M_Q^\dagger &= \begin{pmatrix} \mathbf{m}_q \mathbf{m}_q^\dagger + \epsilon_q \epsilon_q^\dagger & \epsilon_q m_H \\ m_H \epsilon_q^\dagger & m_H^2 \end{pmatrix}, \\ M_Q^\dagger M_Q &= \begin{pmatrix} \mathbf{m}_q^\dagger \mathbf{m}_q & \mathbf{m}_q^\dagger \epsilon_q \\ \epsilon_q^\dagger \mathbf{m}_q & m_H^2 + \epsilon_q^\dagger \epsilon_q \end{pmatrix} \end{aligned} \quad (14)$$

with $m_H = (m_b, m_t)$. Because of $m_H \gg \epsilon_q, m_q$, it is a good approximation to take $V_{R(L)}^Q \approx 1 + \Delta_{R(L)}^Q$. Furthermore, from Eq. (14), one observes that the off-diagonal elements of $M_Q M_Q^\dagger$ are much larger than those of $M_Q^\dagger M_Q$ and thus, $\Delta_L^Q \sim O(\epsilon_q/m_H)$ and $\Delta_R^Q \sim O(m_q \epsilon_q/m_H^2)$. As a result, at the leading order approximation the right-handed unitary matrices could be taken as identity matrices. Consequently, we obtain

$$(\Delta_L^Q)_{i3} = -(\Delta_L^{Q*})_{3i} \approx -\frac{\epsilon_{qi1}}{m_H} = -\frac{\epsilon_{i3}^Q}{\lambda_Q}. \quad (15)$$

It is clear that the induced FCNCs at tree level due to the Yukawa terms are suppressed by $\epsilon_{i3}^Q/\lambda_Q$. Although we cannot get a simple relation for $(\Delta_L^Q)_{ij}$ with $i, j < 3$, the FCNCs involving the first two generations at tree level will be suppressed by the heavy masses of $\phi_{d,s,u}$. The detailed analysis on the neutral Higgs exchange can be found in Ref. [4].

In order to demonstrate that the neutral Higgs mediated FCNC effects will not impose a further serious constraint on the parameters for the charged Higgs, below we give an explicit discussion on the $B_q-\bar{B}_q$ mixing. According to Eq. (7), the relevant Yukawa terms are given by

$$\mathcal{L}_Y = -\bar{Q}'_{Li} Y_{Diq} q'_R \Phi_{q'} + \text{H.c.}, \quad (16)$$

where q' denotes the flavor of d -, s -, and b -quark. Because of Φ_b being the next lightest scalar, in terms of mass eigenstates the dominant effects for FCNCs at tree level in the B processes are written by

$$\mathcal{L}_{\Delta B=1} = -\bar{d}'_{Li} (V_L^D)_{ij} (Y_D)_{j3} b_R \phi_b^0. \quad (17)$$

From the previous analysis, since the off-diagonal elements of the flavor mixing matrix for the right-handed quark are small, Eq. (17) only involves the flavor matrix of V_L^D . Using Eq. (11) and $V_L^D \approx 1 + \Delta_L^D$, we see that

$$\begin{aligned} (V_L^D)_{ij} (Y_D)_{j3} &\approx \lambda^D \delta_{ij} \delta_{j3} + \delta_{ij} \epsilon_{j3}^D + \lambda^D (\Delta_L^D)_{ij} \delta_{j3} \\ &+ O(\epsilon^D). \end{aligned} \quad (18)$$

Furthermore, with the result of $\lambda^D (\Delta_L^D)_{j3} \approx -\epsilon_{j3}^D$ shown in Eq. (15), we find that the ϕ_b^0 -mediated FCNCs not only are associated with the parameter ϵ^D , but also appear in $(\epsilon^D)^2$. As a result, the contributions to the $B_q-\bar{B}_q$ mixing are proportional to $(\epsilon^D)^4/m_{\phi_b^0}^2$. Clearly, by choosing some suitable small value of ϵ^D and $m_{\phi_b} \sim \text{TeV}$, they could be smaller than the current data. In other words, the neutral Higgs mediated $\Delta B = 2$ processes will not provide a further constraint on the parameters for the charged Higgs related effects.

Now, we only pay attention to the charged Higgs related effects. With the physical eigenstates of quarks, the charged Higgs interactions to quarks can be found in Eq. (7), given by

$$\mathcal{L}_{H^+} = -\bar{u}_L V_{\text{CKM}} [V_L^D Y_D] \Phi_{\mathbf{D}}^+ d_R + \bar{u}_R \Phi_{\mathbf{U}}^+ [V_L^U Y_U]^\dagger V_{\text{CKM}} d_L + \text{H.c.}, \quad (19)$$

where $\Phi_{\mathbf{F}}^+$ is a 3×3 matrix and its definition is similar to Eq. (8). We note that $\Phi_{\mathbf{F}}^+$ does not represent the physical charged Higgs scalars. Since there are six Higgs doublets in the model, basically we have five physical charged Higgs scalars and one charged Goldstone boson, which is usually chosen to be

$$G^+ = \sum_{f=t,b,c,s,u,d} \frac{v_f}{v} \phi_f^+ \quad (20)$$

with $v = (\sum_f v_f^2)^{1/2}$. Therefore, to study the effects of physical charged Higgses, in general, one needs to consider a 6×6 mass matrix for these charged scalar fields. According to our earlier analysis, the hierarchy of quark masses is represented by the hierarchy of VEVs of the scalar fields. Because of $v_t \gg v_{f \neq t}$, it should be a good approximation to take $v \approx \sqrt{v_t^2 + v_b^2 + v_c^2} = \sqrt{2(m_t^2 + m_b^2 + m_c^2)^{1/2}}$, i.e., ϕ_t^+ almost aligns to the Goldstone boson. Then, the lightest charged-Higgs will be the ϕ_b^+ . Moreover, since $M_b < M_c \ll M_s \ll M_{d,u}$, the scalar mixing effects associated with $\phi_{s,d,u}^+$ could be neglected due to the suppression of their heavy masses. Based on the character of the PH model, the interesting effects of the charged Higgses are in fact only associated with ϕ_t^+ , ϕ_b^+ and ϕ_c^+ . Effectively, the charged Higgs mass matrix is a 3×3 matrix, which is similar to that in the Weinberg three-Higgs-doublet model [7]. Interestingly, if we further neglect the effect of ϕ_c^+ , the situation returns to the conventional two-Higgs-doublet model [8]. By using Eqs. (11) and (15), we obtain that $\text{diag}(V_L^Q Y_Q) \approx (1, 1, 1)$. Moreover, from Eq. (19), we find that the sizable effects due to the charged Higgs scalars are related to $\bar{t}_L b_R$ and $\bar{t}_R q_L$, where the vertex for the former is given by $\sum_{k=1}^3 (V_{\text{CKM}})_{3k} \times [V_L^D Y_D]_{k3}$ while the latter $\sum_{k=1}^3 [V_L^U Y_U]_{3k}^\dagger (V_{\text{CKM}})_{kq}$. It has no doubt that the coupling $\sum_{k=1}^3 (V_{\text{CKM}})_{3k} [V_L^D Y_D]_{k3}$ is dominated by $k=3$. However, it is more complicated for the coupling $\sum_{k=1}^3 [V_L^U Y_U]_{3k}^\dagger (V_{\text{CKM}})_{kq}$. To see it, we take $q=s$ with the sum

$$\begin{aligned} \sum_{k=1}^3 [V_L^U Y_U]_{3k}^\dagger (V_{\text{CKM}})_{ks} &= [V_L^U Y_U]_{31}^\dagger (V_{\text{CKM}})_{us} \\ &\quad + [V_L^U Y_U]_{32}^\dagger (V_{\text{CKM}})_{cs} \\ &\quad + [V_L^U Y_U]_{33}^\dagger (V_{\text{CKM}})_{ts} \\ &\approx \epsilon_{31}^U \lambda + \epsilon_{32}^U + V_{ts}. \end{aligned} \quad (21)$$

In terms of Eq. (15), the CKM matrix can be expressed by $V_{\text{CKM}} = V_L^U V_L^{D\dagger} \approx 1 + \Delta_L^U - \Delta_L^D$. Accordingly, we get $V_{ts} \approx (\Delta_L^U)_{32} - (\Delta_L^D)_{32} = -\epsilon_{32}^U/\lambda_U + \epsilon_{32}^D/\lambda_D$. Thus, in the phenomenological analysis, we can choose a suitable value of $\lambda_{D(U)}$ so that $V_{ts} > \epsilon_{32}^U$. The dominant effect for the vertex of $\bar{t}_L q_R$ could be simplified to be V_{ts} , i.e., the 3-3 element of $[V_L^U Y_U]$ is the main contribution.

In order to compare with the conventional two-Higgs-doublet model, we rewrite Eq. (19) in terms of quark masses and Eq. (10) as

$$\mathcal{L}_{H^+} = -\sqrt{2} \bar{u}_L V_{\text{CKM}} \mathbf{m}_{\mathbf{D}} \mathbf{V}_{\mathbf{D}}^{-1} \Phi_{\mathbf{D}}^+ d_R + \sqrt{2} \bar{u}_R \Phi_{\mathbf{U}}^+ \mathbf{m}_{\mathbf{U}} \mathbf{V}_{\mathbf{U}}^{-1} V_{\text{CKM}} d_L + \text{H.c.} \quad (22)$$

If we take $\mathbf{V}_{\mathbf{D(U)}}^{-1} = \mathbb{1}_{3 \times 3}/v_{d(u)}$ and $\Phi_{\mathbf{D(U)}}^+ = H_{d(u)}^+ \mathbb{1}_{3 \times 3}$, we can easily get the formulas for the charged-Higgs interactions in the two-Higgs-doublet model to be

$$\begin{aligned} \mathcal{L}_{H^+}^{2\text{Higgs}} &= -\sqrt{2} \bar{u}_L V_{\text{CKM}} \mathbf{m}_{\mathbf{D}} d_R \frac{H_d^+}{v_d} \\ &\quad + \sqrt{2} \bar{u}_R \mathbf{m}_{\mathbf{U}} V_{\text{CKM}} d_L \frac{H_u^+}{v_u} + \text{H.c.} \end{aligned} \quad (23)$$

Furthermore, by using the relationships of

$$\begin{aligned} G^+ &= \cos\beta H_d^+ + \sin\beta H_u^+, \\ H^+ &= -\sin\beta H_d^+ + \cos\beta H_u^+ \end{aligned} \quad (24)$$

with $\cos\beta = v_d/v$, $\sin\beta = v_u/v$ and $v = \sqrt{v_d^2 + v_u^2}$, we have

$$\begin{aligned} \mathcal{L}_{H^+}^{2\text{Higgs}} &= (2\sqrt{2}G_F)^{1/2} (-\bar{u}_L V_{\text{CKM}} \mathbf{m}_{\mathbf{D}} d_R \\ &\quad + \bar{u}_R \mathbf{m}_{\mathbf{U}} V_{\text{CKM}} d_L) G^+ \\ &\quad + (2\sqrt{2}G_F)^{1/2} (\tan\beta \bar{u}_L V_{\text{CKM}} \mathbf{m}_{\mathbf{D}} d_R \\ &\quad + \cot\beta \bar{u}_R \mathbf{m}_{\mathbf{U}} V_{\text{CKM}} d_L) H^+. \end{aligned} \quad (25)$$

III. PHENOMENOLOGIES IN B DECAYS

According to the discussions in Sec. II, we know that there is an essential difference in the couplings of the charged Higgs scalars and quarks between the conventional multi-Higgs and PH models. For instance, if we turn off the CKM matrix elements, from Eq. (19) we see clearly that the couplings in the former are directly proportional to the masses of quarks but those in the latter do not involve new free parameters in the leading contributions. In addition, in the former case, there are no intrinsic limits on the charged Higgs masses, whereas in the latter case, the masses have a preceding hierarchy stemmed from Eq. (6). Consequently, we speculate that the lightest charged Higgs scalar with the couplings of order one in the PH model might have interesting phenomenologies in rare decays suppressed in the SM. From Eq. (19), one can

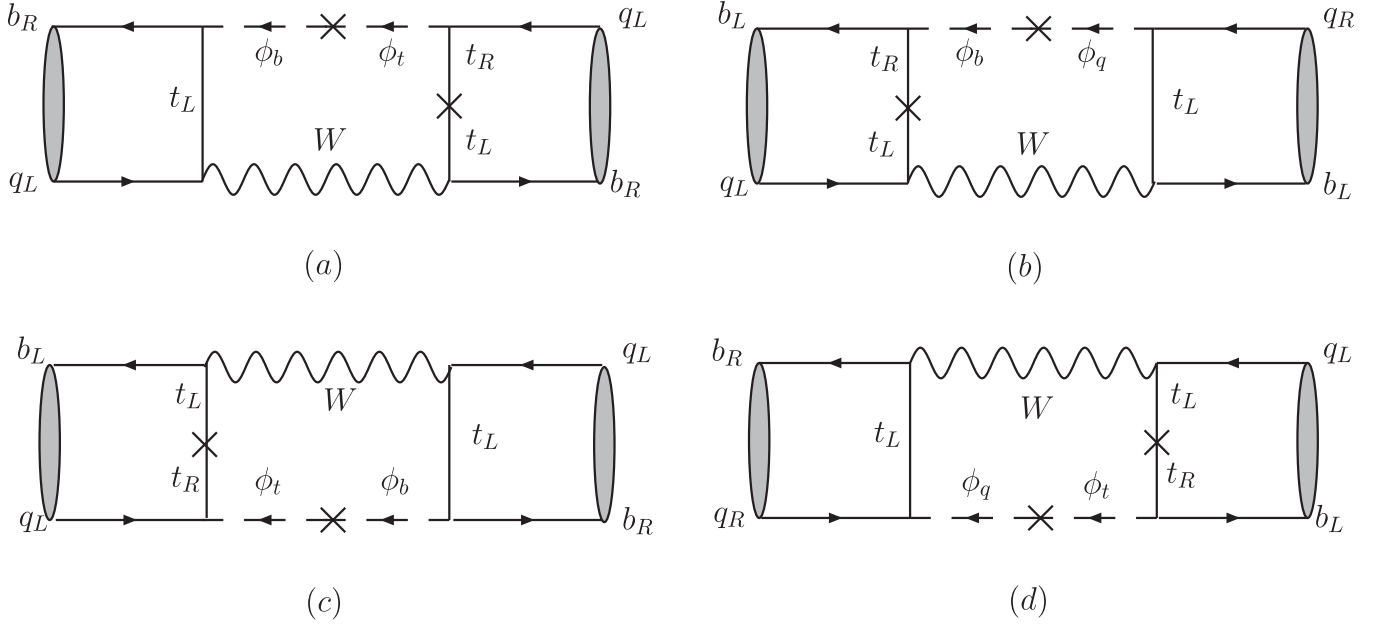


FIG. 1. Box diagrams for the $B_q-\bar{B}_q$ mixing induced by gauge and charged Higgs bosons.

easily find that the large novel effects are associated with t and b quarks and the corresponding charged Higgs scalars are mostly the first two lightest ones of ϕ_t^+ and ϕ_b^+ . Hence, in the following analysis, we will concentrate on the rare B -meson processes involving FCNCs due to the charged Higgs scalars.

A. $B_{s,d}-\bar{B}_{s,d}$ mixings

It is known that all neutral pseudoscalar-antipseudoscalar oscillations in the down type quark systems have been seen. In the SM, since the oscillations are induced from box diagrams, they are ideal places to probe the new physics effects. As mentioned earlier, since $\phi_{s,d}^+$ are much heavier than $\phi_{t,b}^+$, their contributions to the processes in the K -system are small, whereas significant contributions in the B -system could be possible.

To calculate $B_q-\bar{B}_q$ ($q = d, s$) mixings in the PH model, we first consider the diagrams displayed in Fig. 1 due to the gauge and charged Higgs bosons in the loop. The crossed diagrams of internal bosons and fermions are included in the calculations but not explicitly shown up in the figures. To see the mixing effects of the charged Higgs scalars, we present the diagrams in terms of unphysical states.

However, we will formulate the results based on the physical ones. Since Figs. 1(b) and 1(d) involve the heavy charged Higgs ϕ_q^+ , the contributions must be much smaller than those by Figs. 1(a) and 1(c) and therefore, they can be ignored. The effective four-fermion interactions for $\Delta B = 2$ from Figs. 1(a) and 1(c) are given by

$$\begin{aligned} \mathcal{H}_{HW}^a &= -\frac{G_F^2}{2\pi^2} (V_{tq} V_{tb}^*)^2 m_W^2 \left(\frac{m_b m_t}{m_W^2} C_{tb} \right) \\ &\quad \times F(y_t, x_t) \bar{b} \gamma_\mu P_L q \bar{b} \gamma^\mu P_L q, \\ \mathcal{H}_{HW}^c &= \mathcal{H}_{HW}^a \end{aligned} \quad (26)$$

with

$$\begin{aligned} F(a, b) &= \int_0^1 dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \\ &\quad \times \frac{x_1 - x_2 + x_3}{[1 - (1-a)x_1 - (a-b)x_2]^2}, \end{aligned}$$

where $x_t = m_t^2/m_W^2$, $y_t = m_t^2/m_{H^+}^2$, and C_{tb} denotes the unknown mixing element between ϕ_t^+ and ϕ_b^+ . As discussed early, if we regard that the effective charged Higgs scalars are ϕ_t^+ , ϕ_b^+ and ϕ_c^+ , their mixtures are similar to those in the Weinberg's three-Higgs-doublet model. In

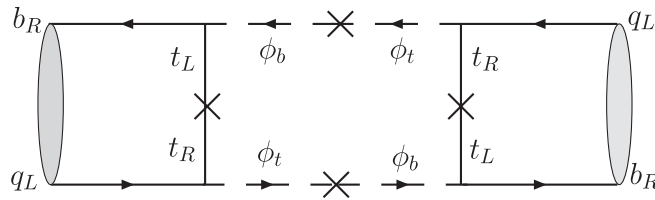


FIG. 2. Box diagrams for $B_q-\bar{B}_q$ mixing arisen from charged scalar bosons.

general, C_{tb} is a complex number. Here, for simplicity, we have only shown the contributions of the lightest physical charged Higgs denoted by H^+ , referred as private charge Higgs. Besides Fig. 1, the diagram in Fig. 2 also yields an important contribution to the mixing. From Eq. (19), we find

$$\mathcal{H}_{HH} = -\frac{G_F^2}{\pi^2} (V_{td} V_{tb}^*)^2 m_W^2 \left(\frac{C_{tb}}{g^2} \frac{m_W}{m_{H^+}} \frac{m_t}{m_{H^+}} \right)^2 \times G(y_t) \bar{b} P_L q \bar{b} P_L q \quad (27)$$

with

$$G(x) = -\frac{2}{(1-x)^2} - \frac{1+x}{(1-x)^3} \ln x.$$

To examine the B_q oscillating effect, we parametrize the matrix elements as [9]

$$\begin{aligned} \langle B_q | (\bar{q}b)_{V-A} (\bar{q}b)_{V-A} | \bar{B}_q \rangle &\approx \frac{4}{3} f_{B_q}^2 \hat{B}_q m_{B_q}, \\ \langle B_q | (\bar{q}b)_{S+P} (\bar{q}b)_{S+P} | \bar{B}_q \rangle &\approx -\frac{5}{6} f_{B_q}^2 \hat{B}_q m_{B_q}, \end{aligned} \quad (28)$$

where $(\bar{q}b)_{V-A} = \bar{q} \gamma_\mu (1 - \gamma_5) b$, $(\bar{q}b)_{S+P} = \bar{q} (1 + \gamma_5) b$, and f_{B_q} is the decay constant of B_q . Accordingly, the $\bar{B}_q \rightarrow B_q$ matrix elements of \mathcal{H}_{HW} and \mathcal{H}_{HH} are given by

$$M_{12}^{qHW(H)} = \frac{G_F^2 m_W^2}{12 \pi^2} (V_{tq}^* V_{tb})^2 f_{B_q}^2 \hat{B}_q m_{B_q} X_{HW(H)}, \quad (29)$$

where

$$\begin{aligned} X_{HW} &= -4 \frac{m_b m_t}{m_W^2} C_{tb} F(y_t, x_t), \\ X_{HH} &= \frac{5}{2} \left(\frac{C_{tb}}{g^2} \frac{m_W m_t}{m_{H^+}} \right)^2 G(y_t), \end{aligned} \quad (30)$$

with g the gauge coupling of $SU(2)_L$. We note that because X_{HW} has the suppression factor of $m_b/m_W F(y_t, x_t)$, it is much smaller than X_{HH} . In the following analysis, we will neglect the contribution of X_{HW} .

To study the influence of new physics on the time-dependent CP asymmetry (CPA), we write the $\bar{B}_q \rightarrow B_q$ transition by combining results from the SM and new physics as

$$M_{12}^q = A_{12}^{qSM} e^{-2i\beta_q} + A_{12}^{qNP} e^{2i(\theta_q^{NP} - \beta_q)} \quad (31)$$

where $\beta_q \equiv \arg(-V_{tq} V_{tb}^*/V_{cq} V_{cb}^*)$ is the weak CP phase of the SM, θ_q^{NP} corresponds to the new CP phase in the PH model and A_{12}^{qSM} is given by

$$A^{qSM} = \frac{G_F^2 m_W^2}{12 \pi^2} (V_{tq}^* V_{tb})^2 f_{B_q}^2 \hat{B}_q m_{B_q} \eta_B S_0(x_t) \quad (32)$$

with $\eta_B \approx 1$ and $S_0(x_t) \approx 0.784 x_t^{0.76}$. Because of $\Delta \Gamma_q \ll \Delta m_q$ in the B -system [1], the time-dependent CPA is found to be

$$\begin{aligned} -S_{J/\Psi M_q} &\simeq \text{Im} \left(\sqrt{\frac{M_{12}^{q*}}{M_{12}^q}} \right) = \sin(2\beta_q - \phi_q^{NP}), \\ \phi_q^{NP} &= \arctan \left(\frac{r_q \sin 2\theta_q^{NP}}{1 - r_q \cos 2\theta_q^{NP}} \right) \end{aligned} \quad (33)$$

with $M_{d(s)} = K_S(\phi)$ and $r_q = A_{12}^{qNP}/A_{12}^{qSM}$. From Eqs. (29) and (30), one gets that $\theta_q^{NP} \equiv \theta_{H^+} = \arg(C_{tb})$ and

$$r_q \equiv r_H = \frac{|X_{HH}|}{\eta_B S_0(x_t)}, \quad (34)$$

which is independent of q in the PH model. From Eq. (33), it is readily seen that the magnitude of ϕ_s^{NP} is controlled by r_H .

B. $b \rightarrow q\gamma$ decays

It is known that $b \rightarrow q\gamma$ decays provide strong constraints on the penguin contributions from new physics. In this subsection, we examine these decays in the PH model. As an illustration, we present the possible dominant effects in Fig. 3. From the figure, we see clearly that Figs. 3(a) [3(c)] and 3(b) [3(d)] involve chirality flip of b [t] and the mixing of ϕ_b^+ and ϕ_q^+ [ϕ_t^+]. Because of $m_b \ll m_t$ and the mixing effect of ϕ_b^+ and ϕ_q^+ ($\propto \gamma_{qb}/M_{\phi_q}^2$) being much smaller than that of ϕ_b and ϕ_t ($\propto \gamma_{tb}/m_W^2$), the contributions of Figs. 3(a) and 3(b) are much smaller than those of Figs. 3(c) and 3(d). Therefore, to study the leading effects, the results of Figs. 3(a) and 3(b) can be neglected. Furthermore, if we replace photons in Fig. 3 with gluons, gluonic penguins can be also generated by the charged Higgs scalars in the PH model.

From Figs. 3(c) and 3(d), we conclude that the effective operators from the charged scalars have the same structures as those in the SM. In order to include the SM contributions, we write the effective Hamiltonian for $b \rightarrow q\gamma$ as [10]

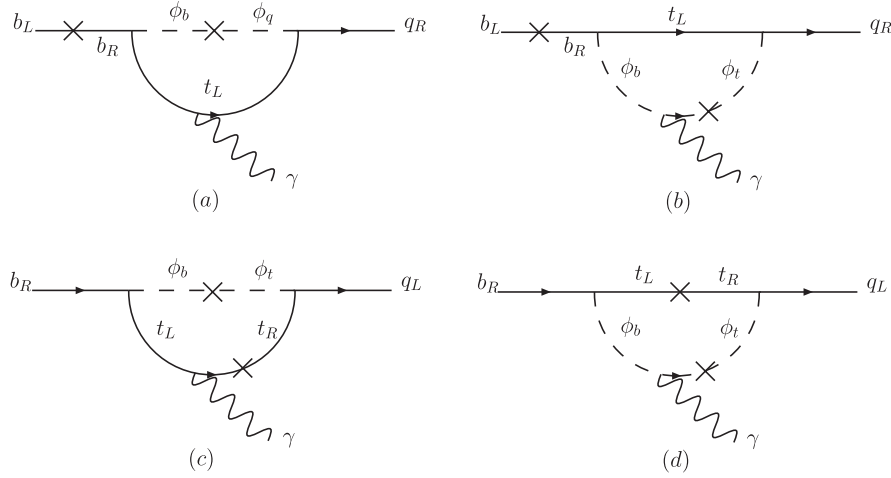


FIG. 3. Penguin diagrams for $b \rightarrow q\gamma$ decays by charged Higgs scalars in the PH model.

$$\mathcal{H}(b \rightarrow q\gamma) = -\frac{G_F}{\sqrt{2}} V_{iq}^* V_{tb} \left[\sum_{i=1}^6 C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8G}(\mu) O_{8G}(\mu) \right], \quad (35)$$

where $O_i(\mu)$ are the effective operators at μ scale and $C_i(\mu)$ are the corresponding Wilson coefficients. Because the dominant effects of the SM are from the terms with C_2 , $C_{7\gamma}$, and C_{8G} , we only show the associated operators of

$$O_2 = (\bar{q}c)_{V-A}(\bar{c}b)_{V-A}, \quad (36)$$

$$O_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q} \sigma^{\mu\nu} (1 + \gamma_5) b F_{\mu\nu},$$

$$O_{8G} = \frac{g_s}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a, \quad (37)$$

respectively, where $(\bar{f}f')_{V-A} = \bar{f} \gamma_\mu (1 - \gamma_5) f'$, e is the electric charge, g_s is the strong coupling constant, α and β denote the color indices, $T_{\alpha\beta}^a$ with $a = 1, \dots, 8$ are the generators of the $SU(3)_C$ gauge symmetry and $F_{\mu\nu}$ ($G_{\mu\nu}^a$) is the electromagnetic (gluonic) field strength. The effective Wilson coefficients by combining the contributions of the W -boson and lightest charged Higgs are given by

$$C_{7\gamma,8G} = C_{7\gamma,8G}^W + C_{7\gamma,8G}^{H^+} \quad (38)$$

with

$$C_{7\gamma}^{H^+} = \frac{v^2}{4m_{H^+}^2} \frac{m_t}{m_b} C_{tb}(Q_t I_c(y_t) + I_d(y_t)), \quad (39)$$

$$C_{8G}^{H^+} = \frac{v^2}{4m_{H^+}^2} \frac{m_t}{m_b} C_{tb} I_d(y_t),$$

where $C_{7\gamma(8G)}^W$ denotes the SM result, Q_t is the electric charge of the top quark and the loop integrals I_c and I_d come from Figs. 3(c) and 3(d), given by

$$I_c(x) = -\frac{3-x}{2(1-x)^2} - \frac{1}{(1-x)^3} \ln x, \quad (40)$$

$$I_d(x) = \frac{1+x}{2(1-x)^2} + \frac{x}{(1-x)^3} \ln x,$$

respectively.

C. $B_q \rightarrow \ell^+ \ell^-$ and $B \rightarrow (P, V) \ell^+ \ell^-$ decays

In this subsection, we discuss the leptonic $B_q \rightarrow \ell^+ \ell^-$ and semileptonic $B \rightarrow (P, V) \ell^+ \ell^-$ decays. The effective Hamiltonian for $b \rightarrow q \ell^+ \ell^-$ in the SM is given by [10–12]

$$\mathcal{H}(b \rightarrow q \ell^+ \ell^-) = -\frac{G_F \alpha}{\sqrt{2} \pi} \lambda_t^q [H_{1\mu} L^\mu + H_{2\mu} L^{5\mu}] \quad (41)$$

with

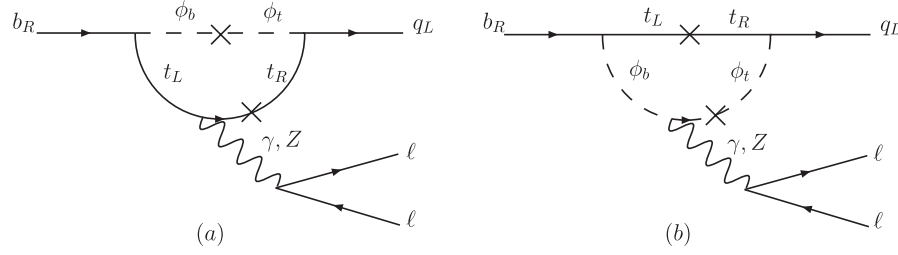
$$H_{1\mu} = C_9^{\text{eff}}(\mu) \bar{q} \gamma_\mu(\mu) P_L b - \frac{2m_b}{k^2} C_{7\gamma}^W(\mu) \bar{q} i \sigma_{\mu\nu} k^\nu P_R b, \quad (42)$$

$$H_{2\mu} = C_{10} \bar{q} \gamma_\mu P_L b,$$

$$L^\mu = \bar{\ell} \gamma^\mu \ell,$$

$$L^{5\mu} = \bar{\ell} \gamma^\mu \gamma_5 \ell,$$

where $\lambda_t^q = V_{iq}^* V_{tb}$, k^2 is the invariant mass of the lepton pair and $C_9^{\text{eff}}(\mu)$, C_{10} and $C_{7\gamma}^W(\mu)$ are the Wilson coefficients (WCs) with their expressions for next-leading-order corrections in Ref. [10]. Since the operator associated with C_{10} is not renormalized under QCD, it is the only one with the μ scale free. In addition, by considering the effects

FIG. 4. Penguin diagrams for $b \rightarrow q \bar{\ell} \ell$ decays generated by the charged scalars in the PH model.

from the one-loop matrix elements of $O_1 = \bar{s}_\alpha \gamma^\mu P_L b_\beta \bar{c}_\beta \gamma_\mu P_L c_\alpha$ and $O_2 = \bar{s} \gamma^\mu P_L b \bar{c} \gamma_\mu P_L c$, the resultant effective WC of C_9 is [10]

$$C_9^{\text{eff}} = C_9(\mu) + (3C_1(\mu) + C_2(\mu))h(x, s),$$

$$h(z, s) = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2} \begin{cases} \ln \left| \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \right| - i\pi, & \text{for } x \equiv 4z^2/s < 1, \\ 2 \arctan \frac{1}{\sqrt{x-1}}, & \text{for } x \equiv 4z^2/s > 1 \end{cases} \quad (43)$$

with $z = m_c/m_b$ and $s = k^2/m_B^2$. Similar to the SM, electroweak penguin diagrams in Fig. 4 mediated by the private charged Higgs scalars can also contribute to $b \rightarrow q \ell^+ \ell^-$. Therefore, in terms of Eq. (19) and the mixture of ϕ_t^+ and ϕ_b^+ , the results of Z- and γ -penguin are formulated to be

$$\mathcal{H}_{a+b}^Z = \frac{G_F \alpha}{\sqrt{2} \pi} \lambda_t^q \{ -X_1^Z \bar{q} \gamma_\mu P_L b [C_V^\ell \bar{\ell} \gamma^\mu \ell - C_A^\ell \bar{\ell} \gamma^\mu \gamma_5 \ell] + X_2^Z \bar{q} P_R b [C_V^\ell \bar{\ell} P_b \ell - C_A^\ell \bar{\ell} P_b \gamma_5 \ell] \},$$

$$\mathcal{H}_{a+b}^\gamma = \frac{G_F \alpha}{\sqrt{2} \pi} \lambda_t^q \left\{ Y^\gamma \bar{q} \gamma_\mu P_L b \bar{\ell} \gamma^\mu \ell + C_{7\gamma}^{H^+} \frac{2m_b}{k^2} \bar{q} i \sigma_{\alpha\beta} k^\beta P_R b \bar{\ell} \gamma^\alpha \ell \right\}, \quad (44)$$

respectively, with

$$X_1^Z = \frac{C_{ib} m_b}{8\pi\alpha} \frac{m_t}{m_H^2} \left[(C_V^t + C_A^t) K_1(y_t) + \frac{1}{2} (C_V^t - C_A^t) I_c(y_t) \right],$$

$$X_2^Z = \frac{C_{ib}}{4\pi\alpha} \frac{m_t}{m_H^2} (C_V^t I_c(y_t) + \cos 2\theta_W I_d(y_t)),$$

$$Y^\gamma = \frac{C_{ib}}{2} \frac{v^2}{m_{H^+}^2} \frac{m_b m_t}{k^2} (1 - Q_t) I_d(y_t),$$

$$C_V^f = T_f^3 - 2 \sin^2 \theta_W Q_f, \quad C_A^f = T_f^3$$

and

$$K_1(x) = -\frac{1-3x}{4(1-x)^2} - \frac{1-x}{2(1-x)^3} \ln x, \quad (46)$$

where T_f^3 is the third component of weak isospin and Q_f is the electric charge of f .

With the effective interactions in Eqs. (41) and (44) for $b \rightarrow q \ell^+ \ell^-$, the BR for the two-body decay $B_q \rightarrow \ell^+ \ell^-$ is straightforwardly given by

$$\mathcal{B}(B_q \rightarrow \ell^+ \ell^-) = \mathcal{B}^{\text{SM}}(B_q \rightarrow \ell^+ \ell^-) \times \left| 1 + \frac{C_A^\ell X_1^Z}{C_{10}} + \frac{C_A^\ell X_2^Z}{C_{10}} \frac{m_{B_q}^2}{m_b} \right|^2 \quad (47)$$

with

$$\mathcal{B}^{\text{SM}}(B_q \rightarrow \ell^+ \ell^-) = \tau_{B_q} \frac{G_F^2 \alpha^2}{16\pi^3} |\lambda_t^q|^2 m_{B_q}^2 f_{B_q}^2 m_\ell^2 \times \left(1 - \frac{4m_\ell^2}{m_{B_q}^2} \right)^{1/2} |C_{10}|^2.$$

Since the BR is proportional to the lepton mass, obviously, the related decays are chiral suppressed. In addition, we see that only the H^+ mediated Z-penguin has the contribution to the decays. In order to study $B \rightarrow (P, V) \ell^+ \ell^-$, we have to know the information on the transition elements of $B \rightarrow (P, V)$ with various transition currents. As usual, we parametrize the relevant form factors as follows:

$$\begin{aligned}
 \langle \bar{P}(p_P) | V_\mu | \bar{B}(p_B) \rangle &= f_+(k^2) \left\{ P_\mu - \frac{P \cdot k}{k^2} k_\mu \right\} + \frac{P \cdot k}{k^2} f_0(k^2) k_\mu, \\
 \langle \bar{P}(p_P) | T_{\mu\nu} k^\nu | \bar{B}(p_B) \rangle &= \frac{f_T(k^2)}{m_B + m_P} \{ P \cdot k k_\mu - k^2 P_\mu \}, \\
 \langle \bar{V}(p_V, \epsilon) | V_\mu | \bar{B}(p_B) \rangle &= i \frac{V(k^2)}{m_B + m_V} \epsilon_{\mu\alpha\beta\rho} \epsilon^{*\alpha} P^\beta k^\rho, \\
 \langle \bar{V}(p_V, \epsilon) | A_\mu | \bar{B}(p_B) \rangle &= 2m_V A_0(k^2) \frac{\epsilon^* \cdot k}{k^2} q_\mu + (m_B + m_V) A_1(k^2) \left(\epsilon_\mu^* - \frac{\epsilon^* \cdot k}{k^2} k_\mu \right) - A_2(k^2) \frac{\epsilon^* \cdot k}{m_B + m_V} \left(P_\mu - \frac{P \cdot k}{k^2} k_\mu \right), \\
 \langle \bar{V}(p_V, \epsilon) | T_{\mu\nu} k^\nu | \bar{B}(p_B) \rangle &= -iT_1(k^2) \epsilon_{\mu\alpha\beta\rho} \epsilon^{*\alpha} P^\beta k^\rho, \\
 \langle \bar{V}(p_V, \epsilon) | T_{\mu\nu}^5 k^\nu | \bar{B}(p_B) \rangle &= T_2(k^2) (\epsilon_\mu^* P \cdot k - \epsilon^* \cdot k P_\mu) + T_3(k^2) \epsilon^* \cdot k \left(k_\mu - \frac{k^2}{P \cdot k} P_\mu \right), \tag{48}
 \end{aligned}$$

where $(V_\mu, A_\mu, T_{\mu\nu}, T_{\mu\nu}^5) = \bar{q}(\gamma_\mu, \gamma_\mu \gamma_5, i\sigma_{\mu\nu}, i\sigma_{\mu\nu} \gamma_5)b$, $m_{B,P,V}$ are the masses of B , pseudoscalar and vector mesons, $P = p_B + p_{P(V)}$, respectively, $k = p_B - p_{P(V)}$ and $P \cdot k = m_B^2 - m_{P(V)}^2$. By equation of motion, we can have the transition form factors for scalar and pseudoscalar currents as

$$\begin{aligned}
 \langle \bar{P} | \bar{q} b | \bar{B} \rangle &\approx \frac{P \cdot k}{m_b} f_0(k^2), \\
 \langle V | \bar{q} \gamma_5 b | \bar{B} \rangle &\approx -\frac{2m_V}{m_b} \epsilon^* \cdot q A_0(k^2). \tag{49}
 \end{aligned}$$

Here, the light quark mass has been neglected. According to the definitions of the form factors, the transition amplitudes for $B \rightarrow (P, V) \ell^+ \ell^-$ can be written as

$$\mathcal{M}_P = -\frac{G_F \alpha \lambda_t^q}{\sqrt{2}\pi} [m_{97} \bar{\ell} \not{p}_K \ell + m_{10} \bar{\ell} \not{p}_K \gamma_5 \ell] \tag{50}$$

with

$$\begin{aligned}
 m_{97} &= (C_9^{\text{eff}} + C_V^\ell X_1^Z - Y^\gamma) f_+ - \frac{m_B}{2} C_V^\ell C_{H^+}^Z f_0 \\
 &\quad + \frac{2m_b}{m_B + m_P} C_{7\gamma} f_T, \\
 m_{10} &= (C_{10} - C_A^\ell X_1^Z) f_+ + \frac{m_B}{2} C_A^\ell C_{H^+}^Z f_0, \tag{51}
 \end{aligned}$$

and

$$\mathcal{M}_V = -\frac{G_F \alpha \lambda_t^q}{\sqrt{2}\pi} [\mathcal{M}_{1\mu} \bar{\ell} \gamma^\mu \ell + \mathcal{M}_{2\mu} \bar{\ell} \gamma^\mu \gamma_5 \ell] \tag{52}$$

where

$$\begin{aligned}
 \mathcal{M}_{1\mu} &= i\tilde{m}_{97}^1 \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_V^\alpha k^\beta - \tilde{m}_{97}^2 \epsilon_\mu^* + \tilde{m}_{97}^3 \epsilon^* \cdot k p_{V\mu}, \\
 \mathcal{M}_{2\mu} &= i\tilde{m}_{10}^1 \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_V^\alpha k^\beta - \tilde{m}_{10}^2 \epsilon_\mu^* + \tilde{m}_{10}^3 \epsilon^* \cdot k p_{V\mu}, \tag{53}
 \end{aligned}$$

with

$$\begin{aligned}
 \tilde{m}_{97}^1 &= \frac{V}{m_B + m_V} (C_9^{\text{eff}} + C_V^\ell X_1^Z - Y^\gamma) + \frac{2m_b}{k^2} C_{7\gamma} T_1, \\
 \tilde{m}_{97}^2 &= \frac{1}{2} (m_B + m_V) (C_9^{\text{eff}} + C_V^\ell X_1^Z - Y^\gamma) A_1 \\
 &\quad + \frac{1}{2} \frac{2m_b}{k^2} P \cdot k C_{7\gamma} T_2, \\
 \tilde{m}_{97}^3 &= \frac{A_2}{m_B + m_V} (C_9^{\text{eff}} + C_V^\ell X_1^Z - Y^\gamma) + \frac{m_V}{m_B} A_0 C_V^\ell C_{H^+}^Z \\
 &\quad + \frac{2m_b}{k^2} C_{7\gamma} \left(T_2 + \frac{k^2}{P \cdot k} T_3 \right), \\
 \tilde{m}_{10}^1 &= \frac{V}{m_B + m_V} (C_{10} - C_A^\ell X_1^Z), \\
 \tilde{m}_{10}^2 &= \frac{1}{2} (m_B + m_V) A_1 (C_{10} - C_A^\ell X_1^Z), \\
 \tilde{m}_{10}^3 &= \frac{A_2}{m_B + m_V} (C_{10} - C_A^\ell X_1^Z) - \frac{m_V}{m_B} A_0 C_A^\ell C_{H^+}^Z. \tag{54}
 \end{aligned}$$

Here, we only pay attention to the light leptons with the explicit effects of m_ℓ ignored.

To get the decay rate distribution in terms of the dilepton invariant mass k^2 and the lepton polar angle θ , we use the k^2 rest frame in which $p_\ell = E_\ell(1, \sin\theta, 0, \cos\theta)$, $p_H = (E_H, 0, 0, |\vec{p}_H| \cos\theta)$ with $E_\ell = \sqrt{k^2}/2$, $E_H = (m_B^2 - k^2 - m_H^2)/(2\sqrt{k^2})$, and $|\vec{p}_H| = \sqrt{E_H^2 - m_H^2}$. By squaring the transition amplitude in Eq. (50) and including the three-body phase-space factor, the differential decay rate as a function of k^2 and θ for $B \rightarrow P \ell^+ \ell^-$ is given by

$$\begin{aligned}
 \frac{d\Gamma_P}{dk^2 d\cos\theta} &= \frac{G_F^2 \alpha^2 |\lambda_t^q|^2}{2^8 m_B^2 \pi^5} \tilde{p}_P |\vec{p}_P|^2 (k^2 - 4E_\ell^2 \cos^2\theta) \\
 &\quad \times (|m_{97}|^2 + |m_{10}|^2). \tag{55}
 \end{aligned}$$

For $B \rightarrow V \ell^+ \ell^-$, by summing up the polarizations of V with the identity $\sum \epsilon_{V\mu}^*(p) \epsilon_{V\nu}(p) = (-g_{\mu\nu} + p_\mu p_\nu / p^2)$, from Eq. (52) the differential decay rate is found to be

$$\begin{aligned}
\frac{d\Gamma_V}{dk^2 d\cos\theta} &= \frac{G_F^2 \alpha^2 |\lambda_t|^2}{2^8 m_B^2 \pi^5} \tilde{p}_V \left[k^2 |\tilde{p}_V|^2 (k^2 + 4E_\ell^2 \cos^2\theta) (|\tilde{m}_{97}^1|^2 + |\tilde{m}_{10}^1|^2) + \frac{|\tilde{p}_V|^2}{m_V^2} (k^2 - 4E_\ell^2 \cos^2\theta) (|\tilde{m}_{97}^2|^2 + |\tilde{m}_{10}^2|^2) \right. \\
&\quad + 2k^2 (|\tilde{m}_{97}^2|^2 + |\tilde{m}_{10}^2|^2) + \frac{k^2}{m_V^2} |\tilde{p}_V|^4 (k^2 - 4E_\ell^2 \cos^2\theta) (|\tilde{m}_{97}^3|^2 + |\tilde{m}_{10}^3|^2) - 2 \frac{k \cdot p_V}{m_V^2} |\tilde{p}_V|^2 (k^2 - 4E_\ell^2 \cos^2\theta) \\
&\quad \left. \times (\text{Re}(\tilde{m}_{97}^2 \tilde{m}_{97}^{3*}) + \text{Re}(\tilde{m}_{10}^2 \tilde{m}_{10}^{3*})) - 8 |\tilde{p}_V| E_\ell k^2 [\text{Re}(\tilde{m}_{97}^1 \tilde{m}_{10}^{2*}) + \text{Re}(\tilde{m}_{97}^2 \tilde{m}_{10}^{1*})] \cos\theta \right]. \quad (56)
\end{aligned}$$

Here, \tilde{p}_H ($H = p$ or V) represents the spatial momentum of the H meson in the B -meson rest frame, given by $\tilde{p}_H = \sqrt{E_H^2 - m_H^2}$ with $E_H = (m_B^2 + m_H^2 - k^2)/(2m_B)$. The forward-backward asymmetry (FBA) is defined by

$$A_{\text{FB}} = \frac{\int_{-1}^1 \omega(\theta) d\cos\theta d\Gamma/dk^2/d\cos\theta}{\int_{-1}^1 d\cos\theta d\Gamma/dk^2/d\cos\theta} \quad (57)$$

with $\omega(\theta) = \cos\theta/|\cos\theta|$. Since Eq. (55) has no linear term in $\cos\theta$, the FBA for $B \rightarrow P\ell^+\ell^-$ vanishes. Hence, only $B \rightarrow V\ell^+\ell^-$ has a nonvanished FBA, given by

$$\begin{aligned}
A_{\text{FB}}^V(k^2) &= -\frac{1}{d\Gamma/dk^2} \frac{G_F^2 \alpha^2 |\lambda_t|^2}{2^8 m_B^2 \pi^5} \\
&\quad \times \tilde{p}_V [8 |\tilde{p}_V| E_\ell k^2 (\text{Re}(\tilde{m}_{97}^1 \tilde{m}_{10}^{2*}) + \text{Re}(\tilde{m}_{97}^2 \tilde{m}_{10}^{1*}))]. \quad (58)
\end{aligned}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Since the contributions to the processes in the B_q mixing, $B \rightarrow X_s \gamma$, $B_q \rightarrow \ell^+ \ell^-$, and $B \rightarrow (P, V)\ell^+ \ell^-$ by the charged-Higgs scalars have strong correlations, the new free parameters are only m_{H^+} and C_{tb} . On the other hand, we can find constraints among these decays due to experimental data. To comprehend the influence of the new charged Higgs on the rare decays, we in turn investigate the above processes. As an illustration, we only focus on the processes with $\ell = \mu$.

For the B_q mixing, besides the mass difference of two physical B -meson states described by $\Delta m_q = 2|M_{12}^q|$, the time-dependent CPA in Eq. (33) is also an important physical quantity to display the new physics. To do the numerical analysis, we take $f_{B_d} \sqrt{\hat{B}_d} = 0.184$ GeV, $f_{B_s} \sqrt{\hat{B}_s} = 0.221$ GeV, $V_{ts} = -0.04 e^{i\beta_s}$ with $\beta_s = 0.019$ and $V_{td} = 8.2 \times 10^{-3} e^{-i\beta_d}$ with $\beta_s = -0.375$ [13], in which the leading SM results are $\Delta m_d^{\text{SM}} = 0.52$ ps $^{-1}$ and $\Delta m_s^{\text{SM}} = 18.25$ ps $^{-1}$. Accordingly, we present the influence of the private charged Higgs in Fig. 5, where $\phi_q \equiv 2\beta_q - \phi_q^{H^+}$, m_{H^+} is set to be $150 \text{ GeV} \leq m_{H^+} \leq 1 \text{ TeV}$ and C_{tb} and θ_{H^+} have been chosen to satisfy $0.49 \leq \Delta m_d \leq 0.51$ ps $^{-1}$ and $17.31 \leq \Delta m_s \leq 19.03$ ps $^{-1}$ [14,15]. We note that although Δm_d has a very high precise measurement with 0.507 ± 0.055 ps $^{-1}$ [1], since the error from the nonperturbative QCD is large, for theoretical

estimations we take a conservative bound. From the figure, we see clearly that if we only consider the constraints of $\Delta m_{d,s}$, the CP phases extracted from time-dependent CPAs of $B \rightarrow J/\Psi(K_S, \phi)$ have significant deviations from those in the SM.

It is known that the BR for $B \rightarrow X_s \gamma$ not only has been measured well to be $(3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$ [13] but also is consistent with the SM prediction of $(3.29 \pm 0.33) \times 10^{-4}$ [16]. Hence, $B \rightarrow X_s \gamma$ could give a strict constraint on the parameters of new physics. To simply get the bound, we adopt the BR for $B \rightarrow X_s \gamma$ to be [17]

$$\begin{aligned}
\frac{\mathcal{B}(B \rightarrow X_s \gamma)_{E_\gamma > (1-\delta)E_\gamma^{\text{max}}}}{\mathcal{B}(B \rightarrow X_c \ell \bar{\nu})} &= \frac{6\alpha}{\pi f(m_c^2/m_b^2)} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} K_{\text{NLO}}(\delta), \\
K_{\text{NLO}}(\delta) &= \sum_{i,j=2,7\gamma,8G} k_{ij}(\delta) \text{Re}(C_i C_j^*) \\
&\quad + k_{77}^{(1)}(\delta) \text{Re}(C_{7\gamma}^{(1)} C_{7\gamma}^*), \quad (59)
\end{aligned}$$

where δ denotes the fraction of the spectrum above the cut, $E_\gamma^{\text{max}} = m_b/2$, $f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z$ is a phase-space factor, K_{NLO} stands for the next-leading-order effect, $C_{7\gamma}^{(1)}$ is the next-leading-order effect of $C_{7\gamma}$ and the values of k_{ij} and $k_{77}^{(1)}$ are given in Table I. Here, we have only considered the case with $\delta = 0.3$. According to the results in Ref. [17], the relevant Wilson coefficients with charged Higgs contributions are found to be

$$\begin{aligned}
C_{7\gamma} &\approx -0.31 + 0.67 C_{7\gamma}^{H^+} + 0.09 C_{8G}^{H^+}, \\
C_{8G} &\approx -0.15 + 0.70 C_{8G}^{H^+}, \\
C_{7\gamma}^{(1)} &= 0.48 - 2.29 C_{7\gamma}^{H^+} - 0.12 C_{8G}^{H^+}.
\end{aligned} \quad (60)$$

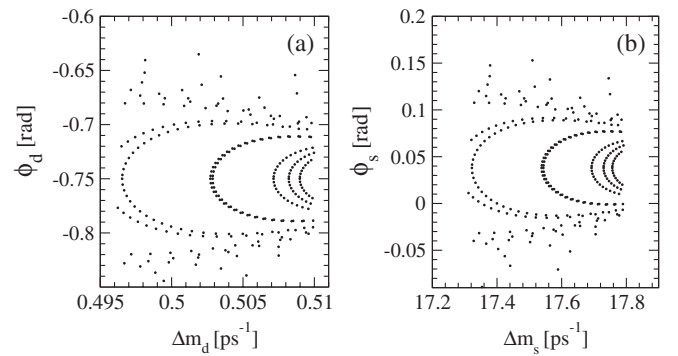


FIG. 5. $\phi_{d[s]} = 2\beta_{d[s]} - \phi_{d[s]}^{H^+}$ versus $\Delta m_{d[s]}$.

TABLE I. Values of k_{ij} (in units of 10^{-2}) with $\delta = 0.3$ [17].

δ	k_{22}	k_{77}	k_{88}	k_{27}	k_{28}	k_{78}	$k_{77}^{(1)}$
0.30	0.11	68.13	0.53	-16.55	-0.01	8.85	3.86

We can also investigate the direct CPA for $B \rightarrow X_s \gamma$, given by [17]

$$\begin{aligned}
 A_{CP}(b \rightarrow s \gamma) &= \frac{\mathcal{B}(\bar{B} \rightarrow X_s \gamma) - \mathcal{B}(B \rightarrow X_s \gamma)}{\mathcal{B}(\bar{B} \rightarrow X_s \gamma) + \mathcal{B}(B \rightarrow X_s \gamma)}, \\
 &= \frac{1}{|C_{7\gamma}|^2} [1.23 \text{Im}(C_2 C_{7\gamma}^*) \\
 &\quad - 9.52 \text{Im}(C_{8G} C_{7\gamma}^*) + 0.10 \text{Im}(C_2 C_{8G}^*)],
 \end{aligned} \tag{61}$$

where the current data is $A_{CP}(b \rightarrow s \gamma) = 0.004 \pm 0.037$ [13]. Since the SM prediction is less than 1% [18], the formula in Eq. (61) has neglected the contributions related to the KM phase. If any sizable CPA is found, it definitely indicates the existence of some new CP violating phases. For $B \rightarrow X_s \gamma$, we first display ϕ_q versus Δm_q in Fig. 6. From the figure, it is clear that the BR of $B \rightarrow X_s \gamma$ has a very serious constraint on $|C_{tb}|$ and m_{H^+} so that the contributions of the private charged Higgs to the time-dependent CPA become very small. To further understand the effects of the charged Higgs on the radiative B decays, we show the correlation between $\mathcal{B}(B \rightarrow X_s \gamma)$ [$A_{CP}(b \rightarrow s \gamma)$] and $|C_{tb}|/m_{H^+}$ in Figs. 7(a) [7(b)]. Interestingly, those values of parameters, which are satisfied with the bound of $\mathcal{B}(B \rightarrow X_s \gamma)$, could still make $A_{CP}(b \rightarrow s \gamma)$ at few percent level where the sensitivity is the same as the current data.

Next, we study the implications of the private charged Higgs on $b \rightarrow q \ell^+ \ell^-$. According to the previous analysis, we learn that $\mathcal{B}(B \rightarrow X_s \gamma)$ and $A_{CP}(b \rightarrow s \gamma)$ could give strong bounds on the free parameters in the PH model. With the constraints, we show the BRs for $B_{s,d} \rightarrow \mu^+ \mu^-$ in Figs. 8(a) and 8(b). From the figures, we see that the contributions in the PH model are very close to

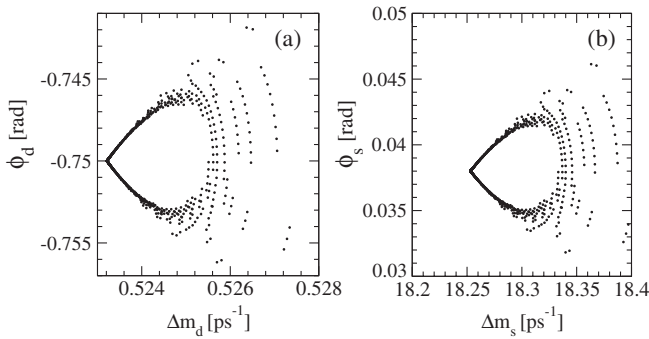


FIG. 6. $\phi_{d[s]} = 2\beta_{d[s]} - \phi_{d[s]}^{H^+}$ versus $\Delta m_{d[s]}$ while the limits of $\mathcal{B}(B \rightarrow X_s \gamma) = (3.52 \pm 0.25) \times 10^{-4}$ and $A_{CP}(b \rightarrow s \gamma) = 0.004 \pm 0.037$ with 1σ errors are considered.

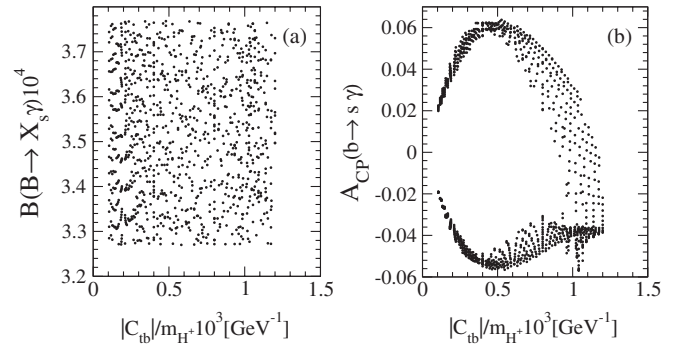


FIG. 7. Correlation between $\mathcal{B}(B \rightarrow X_s \gamma)$ (in units of 10^{-4}) [$A_{CP}(b \rightarrow s \gamma)$] and parameter $|C_{tb}|/m_{H^+}$ (in units of 10^{-3}).

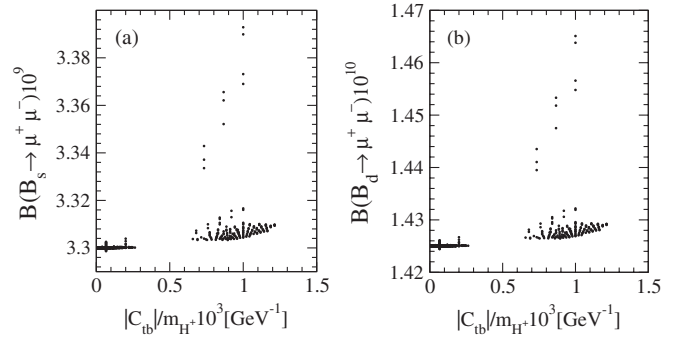


FIG. 8. BR of $B_q \rightarrow \mu^+ \mu^-$ when the constraints of $B \rightarrow X_s \gamma$ are included.

$\mathcal{B}^{\text{SM}}(B_{s(d)} \rightarrow \mu^+ \mu^-) = 3.3(0.14) \times 10^{-9}$ in the SM. Hence, we conclude that the effects of Z-penguin in Fig. 4 are negligible. To estimate the numerical values for $B \rightarrow (P, V) \ell^+ \ell^-$ decays, we use the form factors calculated by the light cone sum rules (LCSRs), parametrized by [19]

$$F(q^2) = \frac{r_1}{1 - q^2/m_1^2} + \frac{r_2}{(1 - q^2/m_2^2)^n} \tag{62}$$

with the associated values of parameters given in Table II and III for $B \rightarrow P$ and $B \rightarrow V$, respectively. From Eqs. (55) and (56) and with the same values of parameters for $B_q \rightarrow \mu^+ \mu^-$, we present the influence of the private charged Higgs on $B^+ \rightarrow (K^+, K^{*+}, \pi^+, \rho^+) \mu^+ \mu^-$ in Fig. 9. We

TABLE II. Values of parameters for $B \rightarrow (K, \pi)$ form factors by LCSRs [19].

$F(q^2)$	r_1	$m_1^2 \text{ GeV}^2$	r_2	$m_2^2 \text{ GeV}^2$	n
$f_+^{B \rightarrow K}$	0.1616	29.3	0.173	29.3	1
$f_0^{B \rightarrow K}$	0.3302	37.46	1
$f_T^{B \rightarrow K}$	0.1614	29.3	0.1981	29.3	1
$f_+^{B \rightarrow \pi}$	0.744	28.3	-0.486	40.73	1
$f_0^{B \rightarrow \pi}$	0.258	33.81	1
$f_T^{B \rightarrow \pi}$	1.387	28.3	-1.134	32.22	1

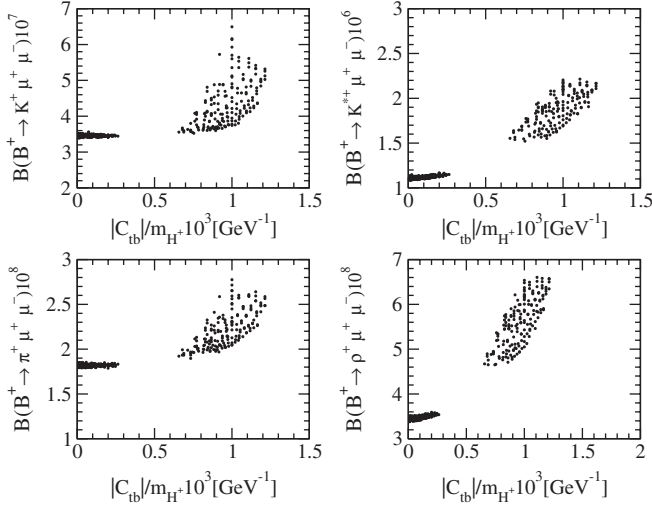


FIG. 9. Correlations of BRs and $|C_{tb}|/m_{H^+}$ for $B^+ \rightarrow (K^+, K^{*+}, \pi^+, \rho^+) \mu^+ \mu^-$.

find that the charged Higgs in the PH model has significant effects on the BRs for $B \rightarrow (P, V) \ell^+ \ell^-$. Since the contributions from H^+ mediated Z -penguin are very small, the main enhancements come from the γ -penguin appearing in $C_{7\gamma}^{H^+}$ of Eq. (39) and Y^γ of Eq. (45). By comparing with the current experimental data, expressed by [1,13]

$$\begin{aligned} \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) &= (4.5_{-0.8}^{+0.9}) \times 10^{-7}, \\ \mathcal{B}(B^+ \rightarrow K^{*+} \mu^+ \mu^-) &= (0.8_{-0.4}^{+0.6}) \times 10^{-6}, \\ \mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) &< 5.1 \times 10^{-8}, \end{aligned} \quad (63)$$

we find that the BR of $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ in the PH model could be larger than the upper value of the current data with 1σ error. In other words, $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ provides a more strict constraint than $B \rightarrow X_s \gamma$ does. We notice that this result relies on the theoretical uncertainty of the non-perturbative $B \rightarrow (P, V)$ form factors. However, the QCD errors could be controlled well with the form factors extracted from the improved measurements on $B \rightarrow K^* \gamma$ and $B \rightarrow (P, V) \ell \nu$ as well as refined lattice calculations. In addition, by a more precise measurement on $B \rightarrow K^* \ell^+ \ell^-$, it also helps to make our conclusion more solid. Hence, the FCNC process of $B \rightarrow K^* \ell^+ \ell^-$ has become an important candidate to constrain the new physics. Finally,

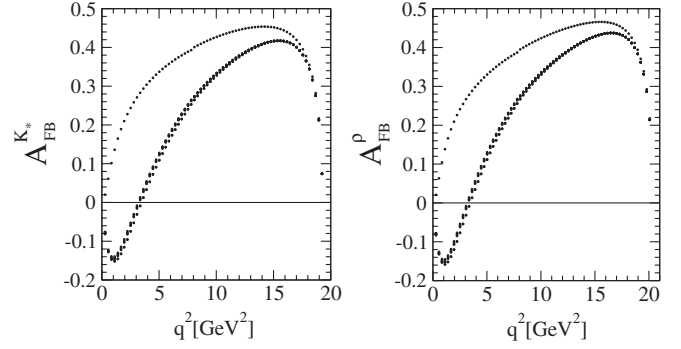


FIG. 10. FBAs for (a) $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ and (b) $B^+ \rightarrow \rho^+ \mu^+ \mu^-$.

by using Eq. (57), we plot the results of the FBA in Fig. 10. It is clear that the shape of the FBA for $B^+ \rightarrow \rho^+ \mu^+ \mu^-$ is the same as that for $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ in the PH model. From the figure, we see that there are two types of curves. The curves crossing the zero point denote the SM-like results in which $C_{7\gamma}$ and $C_{7\gamma}^W$ are the same sign. However, for another type of curves, $C_{7\gamma}$ and $C_{7\gamma}^W$ are opposite in sign. Therefore, to observe the FBA in $B \rightarrow K^* \mu^+ \mu^-$, one can easily judge if the observed $C_{7\gamma}$ has the same sign as that in the SM.

V. SUMMARY

We have studied the charged Higgs effects in the PH model, in which each right-handed quark is associated with one Higgs doublet in the Yukawa sector and the hierarchy of quark masses has been represented by the hierarchy of the Higgs VEVs. It is found that the couplings of the charged Higgs scalars to the fermions are independent of the masses of quarks and order of unity when the CKM matrix elements are excluded. Because of $M_b < M_c \ll M_s \ll M_{d,u}$ of the charged Higgs masses, we have explored the interesting effects of these scalars in B physics. By considering the constraint from the decay of $B \rightarrow X_s \gamma$, the influence of the private charged Higgs on the B_q oscillation is negligibly small. Nevertheless, the CPA of $B \rightarrow X_s \gamma$ could reach the sensitivity of the current data. Moreover, we have found that the BRs of $B \rightarrow (P, V) \ell^+ \ell^-$ are sensitive to the charged Higgs effects. With the form

TABLE III. Values of parameters for $B \rightarrow K^*(\rho)$ form factors by LCSRs [19].

$F(q^2)$	r_1	$m_1^2 \text{ GeV}^2$	r_2	$m_2^2 \text{ GeV}^2$	n
$V^{B \rightarrow K^*(\rho)}$	0.923(1.045)	28.3	-0.511(-0.721)	49.4(38.34)	1
$A_0^{B \rightarrow K^*(\rho)}$	1.364(1.527)	28.3	-0.99(-1.22)	36.78(33.36)	1
$A_1^{B \rightarrow K^*(\rho)}$	0.29(0.24)	40.38(37.51)	1
$A_2^{B \rightarrow K^*(\rho)}$	-0.084(0.009)	52(40.82)	0.342(0.212)	52(40.82)	2
$T_1^{B \rightarrow K^*(\rho)}$	0.823(0.897)	28.3	-0.491(-0.629)	46.31(38.04)	1
$T_1^{B \rightarrow K^*(\rho)}$	0.333(0.267)	41.41(38.59)	1
$T_2^{B \rightarrow K^*(\rho)}$	-0.036(0.022)	48.1(40.88)	0.368(0.246)	48.1(40.88)	2

factors calculated by LCSRs, we have displayed that the constraint from the BR of $B^+ \rightarrow K^{*+} \ell^+ \ell^-$ could be more stringent than that from $B \rightarrow X_s \gamma$. In addition, we have shown that the sign of C_7 in the PH model could be different from the SM and can be further determined by the FBA of $B \rightarrow V \ell^+ \ell^-$.

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